

Name: _____

Date: _____

A2CC: Multiplying Probabilities

Warm Up:

1. The results of a survey of the student body at Central High School about television viewing preferences are shown below.

| | Comedy Series | Drama Series | Reality Series | Total |
|---------|---------------|--------------|----------------|-------|
| Males | 95 | 65 | 70 | 230 |
| Females | 80 | 70 | 110 | 260 |
| Total | 175 | 135 | 180 | 490 |

Are the events “student is a male” and “student prefers reality series” independent of each other? Justify your answer.

Probabilities involving **single-stage experiments** are easy enough because only one thing is happening to affect the probability, i.e. you flip a coin once, you pick one person at random, or you pull one card out of a deck. Probabilities, both empirical and theoretical, become increasingly more complicated with **multi-stage experiments**, where more than one thing happens, i.e. you flip a coin three times. How we handle these types of probabilities actually comes from the conditional probability formula.

Exercise #1: Given that the probability of event B occurring given event A has occurred is

$$P(B | A) = \frac{P(A \text{ and } B)}{P(A)} \text{ answer the following.}$$

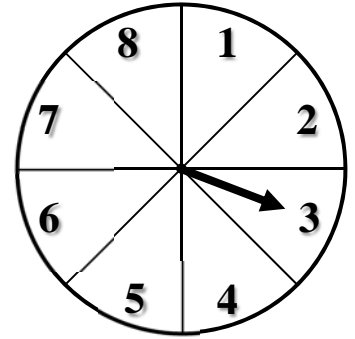
(a) Rewrite this formula, solving for $P(A \text{ and } B)$.

(b) How could you write this formula if events A and B were **independent**?

This rearrangement of the conditional probability formula gives us a useful tool for calculating the probability of events that occur in **multi-stage experiments**. You will easily be able to accomplish this if you systematically phrase the questions as unions of events (events connected by AND).

Exercise #2: Consider the spinner shown below. The spinner is spun twice and the result is recorded.

(a) Are the outcomes of the two spins dependent or independent?



(b) What is the probability that you will get an even on the first spin and a number greater than five on the second spin?

(c) What is the probability that you will spin a prime number and a perfect square (in either order)? Note that this is more complex than (b).

As experiments grow more complicated with more stages, theoretical probability becomes increasingly more complicated. It is especially important to note whether you are sampling **with or without replacement**.

Exercise #3: A class consists of 12 girls and 8 boys. A group of three is picked to give a speech. If the students are picked at random, what is the probability that they all will be boys? Use the events below to show how you calculated your final answer.

Let: E_1 = Event that the first picked was a boy
 E_2 = Event that the second picked was a boy
 E_3 = Event that the third picked was a boy

The **multiplication property of probability** is crucial in many applications in engineering decision making.

Exercise #4: Say that a power generating facility has three primary safety switches in case of an emergency. The probability that any one of these switches would fail is 5%. What is the probability all three will fail given that the switches are **independent** of one another?

Many times when using the multiplication rule we need to be careful about how we frame the question. But, if we properly frame it in terms of AND and OR logical connectors, then the rules of probability will work out.

Exercise #5: A company was determining the effectiveness of its warranty sales on computers. They took data on the number of customers who purchased warranties on two different brands of computers. If a customer was chosen at random, what is the probability they did not purchase a warranty?

| | Percentage of Customers Purchasing | Percent of Those Who Purchased that Also Purchased Warranty |
|--------|------------------------------------|---|
| Type 1 | 68% | 35% |
| Type 2 | 32% | 56% |

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HOMEWORK

APPLICATIONS

1. A fair coin is flipped four times. Find:
 - (a) The probability it will land up heads each time.
 - (b) The probability it will land the same way each time (slightly different from (a)).

2. A first grade class of nine girls and seven boys walks into class in alphabetical order (by last name). What is the probability that three girls are the first to enter the room? Show your calculation.

| | |
|----------|----------|
| (1) 0.15 | (3) 0.35 |
| (2) 0.20 | (4) 0.45 |

3. A bag of marbles contains 12 red marbles, 8 blue marbles, and 5 green marbles. If three marbles are pulled out, find each of the following probabilities. In each we specify either replacement (the marbles go back into the bag after each pull) or no replacement.
 - (a) Find the probability of pulling three green marbles out with replacement.
 - (b) Find the probability of pulling out 3 red marbles without replacement.
- (c) Find the probability of pulling out 3 marbles of the same color without replacement. This is more complex than the other two.

- (d) Find the probability of pulling out two blue marbles and one green marble in any order with replacement. Be careful as there are multiple ways this can be done that will add.

4. The table below shows the percents of graduating seniors who are going to college, broken down into subgroups by gender. If a student was picked at random find the probability that:

(a) They would be a female going to college.

| | Percent of Graduating Seniors | Percent of Subgroup Going to College |
|--------|----------------------------------|---|
| Male | 46% | 78% |
| Female | 54% | 84% |

(b) They would be a male not going to college.

(c) They would be going to college.

(d) They would not be going to college.

5. If a safety switch has a 1 in 10 chance of failing, how many switches would a company want to install in order to have only a 1 in one million chance of them all failing at the same time? Show your reasoning.

REASONING

6. If the probability of winning a carnival game was $\frac{2}{5}$ and Max played it five times, write an expression that would calculate the probability he won the first three games and lost the last two. Use exponents to express your final answer, but do not evaluate.